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Direct numeracy activities and early math skills: Math language as a mediator

Yemimah A. King*, David J. Purpura

Department of Human Development and Family Studies, Hanley Hall, Room 336, Purdue University, 1202 West State St., West Lafayette, IN, 47907-2055

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ABSTRACT

A number of studies provide evidence that the home numeracy environment is important for the development of early numeracy skills. There is also evidence that preschoolers understanding of math language is a strong predictor of numeracy skills. However, there is limited research on the role of math language knowledge in the relation between the home numeracy environment and early numeracy skills. The purpose of this study was to investigate the relation between the home numeracy environment (engagement in direct numeracy activities) and numeracy skills while considering math language as a mediator. Participants included 125 children between 3.12–5.26 years ($M = 4.17$, $SD = 0.58$) and their parents. Parents reported on the frequency of engaging children in direct numeracy activities. Children were assessed in the fall and spring of their preschool year on their numeracy skills and math language knowledge. Results suggest that math language mediates the relation between the direct home numeracy environment and numeracy skills. These findings provide evidence that the relation between the direct home numeracy environment and early numeracy may be explained by preschooler's math language knowledge.

1. Introduction

Children's early math skills are an important predictor of later academic success (Duncan et al., 2007) and set the foundation for later mathematics skills (Blevins-Knabe, 2012; Hill, 2001; Kleemans, Peeters, Segers, & Verhoeven, 2012; LeFevre et al., 2009). A growing body of research has shown that the home numeracy environment (HNE), a construct that encompasses parent-child engagement in numeracy-related activities, predicts preschoolers' numeracy development (Blevins-Knabe & Musun-Miller, 1996; Kleemans et al., 2012; LeFevre et al., 2009). The home numeracy environment may be related to children's early math development as a function of children's understanding of math specific language constructs. As parent-child interactions that fall within the home numeracy environment are often language based (Gunderson & Levine, 2011; Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010; Susperreguy & Davis-Kean, 2016), and children's knowledge of math language is a strong predictor of their early numeracy skills (Purpura & Logan, 2015; Toll & Van Luit, 2014b), it is possible that children's math language knowledge mediates the association between certain components of the home numeracy environment and children's development of numeracy skills.

Given that both math language knowledge and the home numeracy environment are predictors of numeracy skills, it is important to consider that children's understanding of math language may play a role in the association between the home numeracy environment and numeracy skills, which could inform ways that parents may foster children's understanding of math language so that children can benefit from their home numeracy environment. Investigating math language knowledge as a mediator may help explain the relation between the home numeracy environment and children's numeracy skills, as well as, highlight a potential mechanism that underlies how the home numeracy environment is related to early numeracy achievement. Work in this area may contribute to a better understanding of the processes that account for the acquisition of early numeracy and may inform the development of home-based numeracy interventions. The present study will address this gap by investigating the mediating role of prior math language knowledge on the relation between the home numeracy environment and preschooler's later numeracy.

2. Early numeracy

Early numeracy skills, such as counting and identifying quantities, are important for students' later understanding of formal

* Corresponding author.
E-mail address: kingy@purdue.edu (Y.A. King).

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mathematics, such as written addition and subtraction, which are typically learned when children are in school (Baroody, Gannon, Berent, & Ginsburg, 1983). Longitudinal studies show that children with higher numeracy skills during preschool and kindergarten also had higher scores of math achievement in middle and high school (Duncan et al., 2007; Watts, Duncan, Siegler, & Davis-Kean, 2014). Further, some evidence suggests that numeracy skills during preschool predict growth and achievement in math throughout elementary school (Duncan et al., 2007; Geary, 2011; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Watts, Duncan, Clements, & Sarama, 2018).

Beyond students' future academic success, early numeracy skills are related to other life skills as well. For example, early numeracy skills are important for mastering math activities in everyday situations, such as keeping track of time, counting money, and measuring ingredients (NRC, 2009). Understanding the composition and decomposition of quantities during the preschool years is not only important for learning to add and subtract fractions and decimals in school, but is also important for everyday math, such as understanding how to convert a recipe from metric to U.S. standard units (e.g., cups, tablespoons; Blevins-Knabe, 2012). Additionally, numeracy knowledge is necessary for students' career attainment, especially in the science, technology, engineering, and math (STEM) fields (NMAP, 2008). Ultimately, early numeracy skills are not only important for academic success but are also beneficial for everyday problem solving.

3. The relation between language and early numeracy skills

Language skills are important for successfully acquiring early math skills (LeFevre, Polyzoi, Skwarchuk, Fast, & Sowinski, 2010; Sowinski et al., 2015). Notably, many studies have demonstrated a strong relation between children's general language and math skills (Hooper, Roberts, Sideris, Burchinal, & Zeisel, 2010; Purpura & Napoli, 2015; Romano, Babchishin, Pagani, & Kohen, 2010). Prior work has shown that preschoolers' expressive vocabulary is strongly related to most early numeracy skills (Purpura & Ganley, 2014) and that early vocabulary knowledge is significantly predictive of later math achievement (Hooper et al., 2010).

3.1. Early math language

Despite the close relation between general language and numeracy skills, recent evidence indicates that content-specific math language is a stronger predictor of math skills than is general language (Toll & Van Luit, 2014a). Math language is content-specific language that consists of terms used to describe quantitative and spatial relations, but does not include direct references to specific numbers (Purpura & Logan, 2015). Quantitative math language includes words such as "more," "less," "many," and "fewer" (Barner, Chow, & Yang, 2009; Purpura, Logan, Hassinger-Das, & Napoli, 2017). Children must understand quantitative math language to make and describe comparisons between words that describe quantities and groups of objects or numbers. Spatial math language consists of words, such as "near" and "above" that are associated with locations, directions, and shapes (Cannon, Levine, & Huttenlocher, 2007). Children must understand spatial math language to make and describe comparisons between spatial terms and objects' sizes or locations (Pruden, Levine, & Huttenlocher, 2011). In the present study, math language is defined as a child's knowledge of content-specific math terms (e.g., *few*, *some*, and *most*; Purpura, Napoli, & King, 2019; Toll & Van Luit, 2014a & 2014b) and is different from math talk (e.g., parents using number words or counting; *one, two,* and *three*).

Children's math language knowledge in preschool and kindergarten is one of the strongest predictors of their numeracy skills, above and beyond general language knowledge (Purpura & Logan, 2015; Toll & Van Luit, 2014b). Further, Purpura, Napoli, Wehrspann, and Gold (2017) conducted an intervention that provided evidence that increasing preschoolers' exposure to math language by using storybooks resulted in an increase in math language knowledge and numeracy skills. In another study, Toll and Van Luit (2014b) demonstrated that math language mediates the relation between general language and early numeracy skills, suggesting that content-specific math language may be the mechanism by which children use their language knowledge to understand and solve numeracy tasks. Together these studies suggest two key theories: math language may causally underlie numeracy development and children's numeracy skills are highly language-dependent.

4. Children's acquisition of early math

Research findings suggests that young children learn numeracy skills when their parents engage them in math-focused activities at home (Anders, Frazier, & Shallcross, 2012; Baroody & Wilkins, 1999; Blevins-Knabe & Musun-Miller, 1996; Ginsburg, 1977; Manolitsis, Georgiou, & Tziraki, 2013; Melhuish et al., 2008; Niklas & Schneider, 2014; Pan, Gauvain, Liu, & Cheng, 2006; Starkey & Klein, 2000). The home numeracy environment is an everyday context for this early development where parents provide children with resources and activities that may facilitate the development of early numeracy skills such as counting objects, comparing sizes, and understanding terms like "more" and "less" (Blevins-Knabe, 2012). Though the focus on the relation between the home numeracy environment and numeracy achievement has increased in recent years (LeFevre et al., 2009; Manolitsis et al., 2013; Niklas & Schneider, 2014; Thompson, Napoli, & Purpura, 2017), the role that children's math language knowledge may play in this relation is unknown. Understanding the role that math language knowledge may play is particularly important when considering that causal connections between math language knowledge and numeracy skills have been found (Purpura et al., 2017b). Math language knowledge may be an important factor for unterstanding how children benefit from engagement in home numeracy activities. It is important to further investigate the home numeracy environment to understand the role of math language knowledge on the relation between numeracy-related activities and children's early numeracy skills, which contributes to their readiness to learn in school.

5. Home numeracy environment

5.1. Structure of the HNE

The home numeracy environment consists of the frequency of parent-child engagement in direct numeracy activities (e.g., counting, reading number storybooks) and indirect numeracy activities (e.g., measuring ingredients when cooking, talking about money), which support young children's numeracy development (Anderson, 1997; Blevins-Knabe, Austin, Musun, Eddy, & Jones, 2000; Elliott & Bachman, 2018; LeFevre et al., 2009; Manolitsis et al., 2013; Skwarchuk, Sowinski, & LeFevre, 2014; Thompson et al., 2017). Direct numeracy activities are ones in which children are explicitly taught quantitative skills (Skwarchuk et al., 2014). These tasks include identifying names of written numbers, counting objects, and reciting numbers in order. Indirect numeracy activities consist of a broader range of everyday tasks in which children are implicitly taught quantitative skills. Indirect numeracy activities include talking about money, measuring ingredients while cooking, and using numbers to refer to temperatures, time, and dates (Skwarchuk et al., 2014).

5.2. The direct HNE and early numeracy skills

The present study specifically focuses on direct numeracy activities, which have been a more consistent predictor of early numeracy skills than indirect numeracy activities (LeFevre et al., 2010; Manolitsis et al., 2013; Skwarchuk et al., 2014; Thompson et al., 2017), though there are a few exceptions (LeFevre et al., 2009; Skwarchuk et al., 2014). These mixed findings may be due to studies focusing on numeracy skills for different age groups and researchers using different home numeracy environment items. Additionally, the nature of indirect and direct numeracy activities differs, and children's math language knowledge may play a particular mediating role in the relation between the direct numeracy activities and numeracy skills. For example, parents were more likely to provide number comparison and sequencing guidance when completing a direct numeracy task (e.g., number concept tasks) with their child rather than when playing a board game that involved numbers (Vandermaas-Peeler, Boomgarden, Finn, & Pittard, 2012; Vandermaas-Peeler & Pittard, 2014). Additionally, a recent observational study suggests that discussions about numbers rarely occur (e.g. 2% of the time) during parent-child engagement in activities that are often categorized as indirect math activities (e.g., board game, puzzle play, building with Legos; Thippana, Elliott, Gehman, Libertus, & Libertus, 2020). Further, during indirect numeracy activities, parents most often engage their children in counting rather than comparing quantities, which relies more on an understanding of math language (LeFevre et al., 2010; Manolitsis et al., 2013; Thompson et al., 2017). Specifically, young children need to gain understanding of the numeracy-related terms that may be more often used during direct numeracy activities in order to independently problem solve. In contrast, math language knowledge may not be as relevant during indirect numeracy activities where numeracy instruction is incidental and tends to occur less frequently than during direct numeracy activities (Vandermass-Peeler, Nelson, & Bumpass, 2007; Vandermass-Peeler & Pittard, 2014)

According to Vygotsky (1978), children learn from the environment around them, specifically when parents or caregivers facilitate and encourage children's development of specific skills. Early math skills (e.g., numeracy, math language knowledge) are learned within sociocultural contexts where children are actively engaging in numeracy related activities with caregivers or parents (John-Steiner & Mahn, 1996; Miller, 2011). As children develop, they use psychological tools to control, organize, and change their thought or behavior (Miller, 2011). According to Vygotsky (1978), language is the most important psychological tool because comprehending and producing language are processes that transform thinking. Based on this theory, it is hypothesized that children acquire numeracy skills from direct numeracy activities when they have more math language knowledge.

Prior work in this area has demonstrated that the direct home numeracy environment predicts early numeracy skills; however, such work has not yet fully explained this mechanism. Research on the home numeracy environment rarely investigates beyond young children's numeracy skills and it is important to consider the home numeracy environment's relation to children's math language knowledge, which is an important component of early numeracy development (Purpura & Logan, 2015; Purpura et al., 2017b; Toll & Van Luit, 2014). Children may acquire math language from parents during direct numeracy activities and use this language knowledge to evaluate and complete numeracy tasks independently. In order to better understand how the direct home numeracy environment contributes to children's numeracy skills, it is important to consider children's math language knowledge as a mediator, which may inform parents on how to facilitate their children's numeracy development.

Engagement in direct numeracy activities in the home numeracy environment may often rely on the use of math language (e.g., terms used to describe quantities). The home numeracy environment involves language-based interactions and direct numeracy activities have been positively linked to preschoolers' general vocabulary knowledge (Napoli & Purpura, 2018). Additionally, the strongest predictor of math development at the preschool age is math language (Purpura & Logan, 2015; Toll & Van Luit, 2014b). It is likely that as a function of the home numeracy environment—specifically direct numeracy activities—parents may also expose young children to content-specific math language terms that contribute to children's overall understanding of specific numeracy tasks.

Direct numeracy activities that parents and children engage in together frequently involves the use of language to describe, question, and problem solve (NRC, 2009). Use of specific math terms like "more" may help direct their child's thinking, and as their child gains an understanding of the word, he/she may begin to make associations between math language terms, quantities of items, and printed numbers, which will assist in solving independent numeracy tasks. In accordance with Vygotsky's sociocultural theory, math language may be an important psychological tool that is necessary to learn numeracy from direct numeracy activities.

6. Current study

The aim of the current study is to investigate the relation between the home numeracy environment and numeracy skills while considering preschoolers' math language as a mediator. This study includes two hypotheses based on research which suggests that the direct home numeracy environment predicts general language (Napoli & Purpura, 2018) and that both the direct home numeracy environment and math language predicts numeracy skills (Manolitsis et al., 2013; Purpura et al., 2017a). The direct home numeracy environment is expected to predict preschoolers' math language knowledge (H1) and math language is expected to mediate the relation between the direct home numeracy environment and preschoolers' numeracy skills (H2).

7. Method

7.1. Participants

Participants were recruited through convenience sampling from twelve childcare and early education centers. Letters explaining the study, consent forms, and questionnaires were sent home to all parents of 3- to 5-year-old preschool children attending these centers. Parents of 146 students completed the background questionnaire and gave permission for their children to participate. Of those children, 21 were excluded from analyses for one of the following reasons: the child was in kindergarten (some centers provided care for elementary age children and one was connected to a church affiliated elementary school ; $n = 11$), the child left school or moved before testing began ($n = 3$), the child refused to participate in testing ($n = 4$), or the child had language or developmental delays that prevented accurate testing ($n = 3$).

The 125 preschoolers included in the analyses were 54% female, 70% White, 3% Black, 3% Hispanic, 8% Asian, and 15% multiracial, which is approximately representative of the local demographics. Children ranged in age from 3.12 to 5.26 years ($M = 4.17$, $SD = 0.58$) at time of parental consent. Parents reported their level of education by responding to the prompt "What is your highest level of education?" with nine options ranging from "8th grade or less" (1) to "Doctoral/Postgraduate Degree" (9). Parents' highest education ranged from having earned a GED to holding a graduate degree; 26% of parents had some college or less, 32% had an Associate's or

Bachelor's degree, and 42% had a graduate degree. Of the 125 participants, 11 were missing Time 2 numeracy scores and four were missing scores for the home numeracy environment. To address missing data from parent questionnaires and child assessments, FIML was used in StataSE 15, that way all available information was used to estimate the models. Rather than dropping observations with missing data, which would result in a reduction of power, Full Information Maximum Likelihood (FIML) was used in StataSE 15 as a best practice for analyzing incomplete data and using all available information to estimate models (Enders, 2010)

8. Measures

8.1. Early numeracy

The Preschool Early Numeracy Skills Screener – Brief Version (PENS-B; Purpura, Reid, Eiland, & Baroody, 2015) was used to evaluate preschoolers' numeracy skills. The PENS-B is a 24-item measure which takes approximately five minutes to administer and assesses the broad numeracy skills that children are exposed to in preschool and kindergarten. The PENS-B does not include the use of manipulatives, and only the assessment manual is needed to administer the test. The testing binder and scoresheet includes instructions for the researcher as well as a series of pictures that represent math concepts for the child to choose from, count, or compare. Specific assessment areas include set comparison, numeral comparison, one-to-one correspondence, number order, numeral identification, ordinality, and number combinations. Children received one point for each correct answer. Although all 24 items were administered, a ceiling rule was applied during analyses such that children did not receive points for any correct responses after three consecutive incorrect responses (Purpura et al., 2015). The PENS-B has high internal consistency ($\alpha = .92$) and is highly corelated with the Test of Early Mathematics Ability-Third Edition (TEMA-3; $r = .73$). Children were assessed on early numeracy in both the fall and the spring.

8.2. Math language

The math language subtest is a researcher-created measure of mathematics-specific language. The measure has 16 items assessing comparative language (e.g., more, less, take away) and spatial language (e.g., near, far, before). Children were awarded one point for each correct response. In prior work (Purpura & Logan, 2015), these items were selected from a larger battery (i.e., broader range) of items using an item response theory framework. The selected items had a range of difficulty parameters and strong discrimination parameters. The specific words included on this measure were intended to be broadly representative of the quantitative and spatial language associated with mathematics. Quantitative words included take away, a little bit, most, more, fewest, and less. Spatial words included: nearest, under, first, far, below, front, middle, end, last, and before. All items were designed to be completed without exact quantitative skills and in a non-numeracy context. For example, the quantitative questions were asked in different ways: (a) comparing dots with such a gross difference that children would be able to respond correctly regardless of numeracy ability as long as they knew the meaning of the language terms (e.g., 10 vs. 2) and (b) using a picture of mostly full and mostly empty glasses when asking "Which glass has the most water?" or "Which glass has less water?" This math language task had an internal consistency of 0.80 for this sample. Children were assessed on math language in the fall and spring.

8.3. General language

The Test of Preschool Early Literacy (TOPEL; Lonigan, Wagner, Torgesen, & Rashotte, 2007) was used to evaluate preschoolers' general language ability using the definitional vocabulary subtest. This subtest includes 35 items with two questions per item for a total of 70 possible points. Children were asked to provide definitions and explanations of words (e.g., "What is this? What is it for?"). Children were awarded one point for correctly identifying the picture and one point for providing a correct description of the picture. The definitional vocabulary assessment was stopped when the child responded incorrectly to both questions in an item for three consecutive items (i.e., six total questions). This task had an internal consistency of 0.94.

8.4. Home numeracy environment

Parents were asked to complete a researcher-created background information questionnaire. The questionnaire was based on previous research of LeFevre and colleagues (2009). Parents provided socioeconomic status information, such as educational achievement, income, and characteristics of the family and home environment. Parents also reported the frequency of practicing nine indirect numeracy activities and seven direct numeracy activities in the home with their children by responding to the prompt "In the past month, how often did you and your child engage in the following activities?" with six options ranging from "never" (0) to "multiple times per day" (5). Indirect numeracy activities included using calendars and dates, measuring ingredients when cooking, playing board games with die or spinner, using terms more/less, engaging in timed activities, making/sorting collections, sort things by size, color or shape, having your child wear a watch, and talking about money when shopping. Direct numeracy activities included counting objects, printing numbers, reading number storybooks, using number activity books, counting down from 10, learning simple sums, and identifying names of written numbers. The indirect HNE had an internal consistency of 0.77 and the direct HNE had an internal consistency of .72. Parent surveys of HNE were collected prior to preschoolers' first assessment.

There was an average of 4.42 months and range of two to six months between fall (Time 1) assessments and spring (Time 2) assessments of both numeracy skills and math language knowledge. However, only one participant was tested two months apart. Trained graduate and undergraduate research assistants visited children at their childcare and early education centers to administer assessments one-on-one. Additionally, research assistants attended team meetings with the study PI periodically to discuss and address any potential challenges with assessments. All data collectors were trained and/or retrained prior to each data collection period (Time 1 and Time 2).

9. Analytical strategy

Data were analyzed using StataSE 15. A regression analysis and a mediation analysis were conducted to test the first and second hypotheses, respectively. Math language (averaged across Time 1 and 2) was used as the dependent variable in the first model and numeracy skills at Time 2 was used as the dependent variable in the second model. In both models, age, gender, parental education, and general language ability at Time 1 were used as control variables due to established associations with math language and early numeracy (Chang, Sandhofer, & Brown, 2011; Jeon, Buettner, & Hur, 2014; Purpura & Reid, 2016; Sarama & Clements, 2009). A math language average score was created to be used in the regression and mediation analysis to test whether the direct HNE is associated with the preschooler's average level of math language between Time 1 and Time 2. It is recommended to use three separate time points or waves of data when running mediation analyses with longitudinal data (Cole & Maxwell, 2003). The math language average score

Table 1
Descriptive Statistics and Correlations between Variables, Child Measures, and Direct Home Numeracy.

	Descriptive Statistics						Correlations						
	N	*M*	*SD*	Range	Min.	Max.	1	2	3	4	5	6	7
Age	125	4.18	.58	2.14	3.12	5.26	—						
Gender	125	—	—	—	—	—	−.21*	—					
Parent Education	124	6.94	1.67	6.00	3.00	9.00	−.07	−.06	—				
Vocabulary T1	125	50.82	12.35	58.00	8.00	66.00	.57*	−.00	.11	—			
Numeracy T1	125	10.17	5.85	23.00	.00	23.00	.60*	−.00	.35*	.53*	—		
Numeracy T2	114	13.55	5.95	24.00	.00	24.00	.55*	−.02	.19*	.51*	.69*	—	
Math Lang. Avg.	114	12.28	3.11	13.00	4.50	16.00	.52*	.00	.28*	.64*	.66*	.71*	—
Direct HNE	121	2.24	.79	4.29	.14	4.43	.27*	.01	.01	.21*	.22*	.34*	.33*

Note. Minimum and maximum for parent education are 3 = GED and 9 = Doctoral/Postgraduate Degree. Direct HNE = Direct home numeracy environment.
*$p < .05$.

Table 2
Direct HNE Predicting Math Language (Average Score).

Predictor	β	*SE*
Age	.06	.09
Gender	.03	.06
Parent Education	.14*	.07
Vocabulary T1	.38**	.07
Numeracy T1	.35**	.09
Direct HNE	.15*	.06

*$p < .05$, ** $p < .01$.

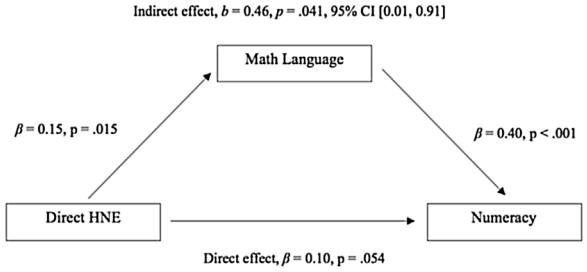

Fig. 1. Model of the direct HNE as a predictor of numeracy skills at Time 2, mediated by average math language knowledge.

was used because it provided a better estimate of children's math language knowledge between Time 1 and Time 2 (NICHD ECCRN & Duncan, 2003). Early numeracy ability at Time 1 was also included as a control variable due to Time 1 numeracy being associated with numeracy ability at Time 2. A series of robustness checks including the indirect HNE in the models were conducted and included as supplementary post hoc analyses in the Appendix.

10. Results

10.1. Descriptive statistics

Means, standard deviations, and ranges for control variables, child measures, and direct HNE scores are presented in Table 1. Additionally, correlations between control variables, numeracy, math language, and the direct HNE are presented in Table 1. Children's age was correlated with vocabulary, numeracy, and math language skills, as well as the direct HNE. Parents' highest level of education was correlated with preschoolers' numeracy and math language. Children's vocabulary scores were correlated with numeracy, math language, and direct HNE. Numeracy and math language, as well as the direct HNE were also strongly correlated. The skewness and kurtosis for all variables were within an acceptable range (between -2 and +2; George & Mallery, 2010). The intraclass correlation coefficient (ICC) for school level variance on the PENS-B was <0.001 in the current study, suggesting that there was very little systematic school-level variation for children's numeracy skills.

10.2. Primary analyses

Results of the regression analysis and mediation model indicated that the direct HNE was a significant predictor of math language, $\beta = 0.15$, $SE = .062$, $p = .015$ (see Table 2 and Fig. 1). Additionally, Fig. 1 illustrates that math language was a significant predictor of numeracy skills, $\beta = 0.40$, $SE = .091$, $p < .001$, which supported both hypotheses. The effect of the direct HNE on numeracy skills attenuated after controlling for the mediator, math language, $\beta = 0.10$, $SE = .054$, $p = .054$, consistent with mediation. The indirect effect was tested using a bootstrap estimation approach with 1000 samples (Shrout & Bolger, 2002). These results indicated the indirect coefficient was significant, $b = 0.46$, $SE = .230$, $p = .041$, 95% CI= [.011, .913].

11. Discussion

This study examined the relation between the direct home numeracy environment and children's numeracy development, and whether children's math language skills mediated that relation. The results suggested that the direct home numeracy environment was related to children's average math language knowledge. Results also indicated that children's math language knowledge mediated the relation between the direct home numeracy environment and numeracy skills. These findings suggest that the direct home numeracy environment is associated with both numeracy skills and math language knowledge and that math language may be a mechanism that explains the relation between the direct home numeracy environment and early numeracy.

11.1. Direct home numeracy environment predicting math language

Results suggest that there is a relation between direct numeracy activities and math language knowledge. Though parent math language use was not measured in this study, parents may use high levels of math language when explaining numeracy concepts to children and it is possible that they use direct numeracy activities to explain these numeracy concepts. Prior work has shown that parents tend to provide more numeracy comparison and sequencing guidance during times where the goal is to explicitly teach math rather than when they are engaging in indirect numeracy activities (Vandermaas-Peeler, Ferretti, & Loving, 2012; Vandermaas-Peeler & Pittard, 2014). This heightened use of parent math language during direct activities may be associated with children's understanding of math language. Although the cur-

rent direct home numeracy environment measure only captures frequency of activity engagement, direct numeracy activities are thought of as activities where children are explicitly taught numeracy skills (Skwarchuk et al., 2014), therefore, parents may use more numeracy-focused language to contribute to young children's understanding during these activities. In contrast, math language knowledge may be less relevant during indirect numeracy activities where parent dialogue is less focused on numeracy concepts outside of counting (Vandermaas-Peeler et al., 2012a, Vandermaas-Peeler, Nelson, & Bumpass, 2007). Future work is needed to explore whether the type of home numeracy activities, direct or indirect, lead to differences in math language use by parents and children, and whether these differences in language use explain later child numeracy skills.

11.2. Math language as a mediator

Mediation analyses investigated whether children's average math language knowledge mediated the relation between the frequency of child engagement in direct numeracy activities and later numeracy skills. Results demonstrated that average math language knowledge mediated the relation between the frequency of engaging in direct numeracy activities and later child numeracy skills. This suggests that the relation between engagement in direct numeracy activities and later numeracy performance is explained by young children's math language knowledge. This finding is in line with previous studies that find math language acts as a mediator of the relation between preschoolers' general language and numeracy skills (Toll & Van Luit, 2014b; Purpura et al., 2017b). This finding supports the idea that children need to understand math language terms when learning early numeracy skills and this applies to learning early numeracy within the direct home numeracy environment.

Results of the current study are also consistent with recent evidence suggesting that children's general language and number word knowledge mediates the relation between parent education and children's math ability (Slusser et al., 2019). Parent education may be related to children's language and number word knowledge through parents linguistic input including their use of number words while communicating with children. Additionally, observational work has provided evidence that parents tend to use number words when communicating with their child during math activity engagement (Thippana et al., 2020). Further, Thippana et al. (2020) found that parent report of math activities in the home is significantly related to their number talk at home. It may also be the case that math language is used during conversations around numbers (to compare, contrast, and describe quantities) especially during direct numeracy activities where number talk is intentional rather than incidental. Our results build on previous explanatory mediation models and suggests that direct home numeracy activities may be a stimulating context for children to build math language knowledge and in turn acquire more numeracy skills.

Given that early math ability is one of the best predictors of later math achievement and academic success (Nguyen et al., 2016; Rittle-Johnson et al., 2017; Watts et al., 2014), having evidence of the potential processes explaining how direct home numeracy activities relate to numeracy development is important for improving family-oriented prevention programs aimed at early numeracy development. Exposure to math language should be embedded within numeracy-related activities to give children the opportunity to build math language knowledge that is necessary for numeracy development (Toll & Van Luit, 2014b). It appears that when parents engage their children in direct numeracy activities, they may expose their children to math language terms that can help children develop numeracy skills. For example, during a direct numeracy activity such as printing numbers, parents may initiate a dialogue with children explaining how some numbers represent quantities that are "bigger", "smaller", "more", or "less" than the quantities of other numbers (Vandermaas-Peeler & Pittard, 2014). Results of the current study fill a gap and provide a rationale for future observational research to investigate exactly how math language knowledge facilitates children's numeracy learning within the context of direct numeracy activities.

12. Limitations and future directions

Although the results of the current study yield important insights regarding the relations between the home numeracy environment, early numeracy skills, and math language knowledge, limitations of the study and areas for future research should be noted. One limitation is that this study design was observational in nature, and therefore, causal implications cannot be inferred. Though causal implications cannot be made, results provide evidence that children's math language knowledge may support their acquisition of numeracy skills from direct numeracy activities that they experience with parents. Prior research provides evidence for a causal relation between math language knowledge and numeracy skills (Purpura et al., 2017b); however, additional research is needed to evaluate whether or not there is a causal relation between the direct home numeracy environment and math language knowledge.

A second limitation is that parent report was used to measure frequency of parent and child engagement in the home numeracy environment, which may have resulted in reporter bias. That is, it is possible that parents may have been influenced by social desirability while completing the survey or may not have remembered how often they engaged with their child in home numeracy activities in the month prior to the study visit. Future research should use real time methods, including time diary approaches, to assess parent report of engagement in daily activities with children so that parents can report on daily activities in the moment rather than trying to remember prior engagement over a month-long period. Additionally, this study focuses on the frequency of direct home numeracy activities rather than a measure of quality, such as the use of math language during these activities. Future research on the home numeracy environment is needed to understand the type of parent involvement that is needed to influence children's acquisition of numeracy skills. More work is needed to examine the use of math language during parent-child practices at home and how that relates to children's development of numeracy skills and math language knowledge. Further studies should include observational methods to measure the content-specific language environment that surrounds engagement in home numeracy activities (Levine et al., 2010). Specifically, more observational work is needed to understand how numbers are discussed both during direct and indirect numeracy activities and why it is important for children to understand math language knowledge in order to gain the most numeracy knowledge during direct numeracy activities.

Another limitation is that the race/ethnicity distribution of the sample was relatively homogeneous. Seventy percent of the sample was Caucasian; therefore, results may not generalize to other racial and ethnic groups. Though recent research shows that the home numeracy environment is a predictor of monolingual Spanish speaking preschooler's numeracy skills (Susperreguy, Douglas, Xu, Molina-Rojas, & LeFevre, 2018), more work is needed to understand the relations between the home numeracy environment, early numeracy, and math language knowledge for young children from more diverse ethnic and linguistic backgrounds. Additionally, future work should investigate the role that school type (e.g., early care or preschool program) and time in school may play in these relations.

13. Conclusion

The findings from this study fill an important gap in the home numeracy environment literature. Math language was found to be a mediator of the relation between direct numeracy activities and numeracy skills. This finding extends findings from previous work indicating significant relations between the direct home numeracy environment and numeracy skills. Particularly, this study's results provide evidence that the direct home numeracy environment is related to math language skills and suggest that math language mediates the relation between the direct home numeracy environment and numeracy skills. Based on these findings, it is plausible that math language may be a valuable target in home-based numeracy interventions; however, more work is needed to explicitly evaluate if supporting the development of math language knowledge can subsequently support the effects of direct numeracy engagement. Additionally, these findings provide rationale for future research to investigate the use of math language during direct numeracy activities to better understand its relations to early math outcomes. Results of this study provide a stepping stone to understand the broader mechanisms by which direct numeracy activities may be linked to children's math development. Understanding these mechanisms is a critical step towards designing and implementing instructional efforts for supporting effective parent-child engagement around math. Better insight on parents' math language use during home numeracy practices is necessary to understand how preschoolers' math language and numeracy learning may be fostered.

CRediT authorship contribution statement

Yemimah A. King: Conceptualization, Formal analysis, Writing - original draft. **David J. Purpura:** Supervision, Writing - review & editing.

Appendix A. Supplementary data

Supplementary material related to this article can be found, in the online version, at doi:https://doi.org/10.1016/j.ecresq.2020.09.012.

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